Q:- 2(a)

$$4x_1 + x_2 + 2x_3 = 16$$
$$2x_1 + 5x_2 + 3x_3 = 19$$
$$3x_1 + 2x_2 - x_3 = 12$$

let us express the above system in following form

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$(A|b) = \left[\begin{array}{ccc|c} 4 & 1 & 2 & 16 \\ 2 & 5 & 3 & 19 \\ 3 & 2 & -1 & 12 \end{array}\right]$$

largest Element in Col- 1 is 4
So that,

$$\left[\begin{array}{ccc|c} 4 & 1 & 2 & 16 \\ 2 & 5 & 3 & 19 \\ 3 & 2 & -1 & 12 \end{array}\right]$$

$R_2 \to 2R_2 - R_1$

$$\left[\begin{array}{ccc|c} 4 & 1 & 2 & 16 \\ 0 & 9 & 4 & 22 \\ 3 & 2 & -1 & 12 \end{array}\right]$$

By Applying $R_3 \to R_3 - \frac{3}{4} R_1$

$$\left[\begin{array}{ccc|c} 4 & 1 & 2 & 16 \\ 0 & 9 & 4 & 22 \\ 0 & 5/4 & -5/2 & 0 \end{array}\right]$$

$R_3 \to 9R_3 - \frac{5}{4} R_2$

$$\left[\begin{array}{ccc|c} 4 & 1 & 2 & 16 \\ 0 & 9 & 4 & 22 \\ 0 & 0 & \frac{-59}{2} & \frac{-55}{2} \end{array}\right]$$

By using back Substitution

$$\frac{-59}{2} x_3 = \frac{-55}{2}$$

$$\boxed{x_3 = 0.932}$$

$$9x_2 + 4 \times 0.932 = 22$$

$$\boxed{x_2 = 2.030}$$

$$4x_1 + 2.030 + 2 \times 0.932 = 16$$

$$\boxed{x_1 = 3.027}$$

Q 2(b) :-

$$\begin{bmatrix} 5 & -5 & -1 \\ 1 & -4 & 1 \\ -2 & 1 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ -4 \\ -18 \end{bmatrix}$$

Rearranging equations

$$5x_1 - 5x_2 - x_3 = -8$$
$$x_1 - 4x_2 + x_3 = -4$$
$$-2x_1 + x_2 - 6x_3 = -18$$

Gauss-Jacobi Scheme is as follows

$$x_1 = \frac{-8 + 5x_2 + x_3}{5} \quad \text{——— (I)}$$

$$x_2 = \frac{-4 - x_1 - x_3}{-4} \quad \text{——— (II)}$$

$$x_3 = \frac{-18 + 2x_1 - x_2}{-6} \quad \text{——— (III)}$$

Taking $x_1 = x_2 = x_3 = 0$

$$x_1 = \frac{-8}{5} = -1.6$$

$$x_2 = \frac{-4}{-4} = 1$$

$$x_3 = \frac{-18}{-6} = 3$$

Second iterations

$$x_1 = \frac{-8+5+3}{5} = 0$$

$$x_2 = \frac{-4+1.6000-3}{-4} = 1.35$$

$$x_3 = \frac{-18+2(-1.6000)-1}{-6} = 3.7$$

Third iteration

$$x_1 = \frac{-8+5(1.35)+3.7}{5} = 0.49$$

$$x_2 = \frac{-4-3.700}{-4} = 1.9250$$

$$x_3 = \frac{-18+2(0.000)-1.3500}{-6}$$

$$= 3.2258$$

Fourth iteration

$$x_1 = \frac{-8+5(1.9250)+3.225}{5}$$

$$= 0.9700$$

$$x_2 = \frac{-4-0.4900-3.225}{-4}$$

$$= 1.9288$$

$$x_3 = \frac{-18+2(0.4900)-1.9250}{-6}$$

$$= 3.1575$$

Fifth iteration

$$x_1 = \frac{-8+5(1.9288)+3.1575}{5} = 0.9603$$

$$x_2 = \frac{-4-0.9700-3.1575}{-4} = 2.0319$$

$$x_3 = \frac{-18+2(0.9702)-1.9290}{-6} = 2.9981$$

$$\begin{bmatrix} 5 & -5 & -1 \\ 1 & -4 & 1 \\ -2 & 1 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ -4 \\ -18 \end{bmatrix}$$

Rearranging equations

$$5x_1 - 5x_2 - x_3 = -8$$
$$x_1 - 4x_2 + x_3 = -4$$
$$-2x_1 + x_2 - 6x_3 = -18$$

Gauss-Seidal Method

$$x_1 = \frac{-8 + 5x_2 + x_3}{5} \quad \text{--- (I)}$$

$$x_2 = \frac{-4 - x_1 - x_3}{-4} \quad \text{--- (II)}$$

$$x_3 = \frac{-18 + 2x_1 - x_2}{-6} \quad \text{--- (III)}$$

Taking $x_1 = 0, x_2 = 0\ x_3 = 0$

$$x_1 = \frac{-8}{5} = -1.6$$

$$x_2 = \frac{-4 + 1.6 - 0}{-4} = +0.6$$

$$x_3 = \frac{-18 + 2 \times -1.6 + 0.6}{-6} = 3.6333$$

2nd Iteration

$$x_1 = \frac{-8 + 5(0.6) + 3.6333}{5}$$
$$= -0.2733$$

$$x_2 = \frac{-4 + 0.2733 - 3.6333}{-4}$$
$$= 1.84$$

$$x_3 = \frac{-18 + 2(-0.2733) - 1.84}{-6}$$
$$= 3.3978$$

3rd Iteration

$$x_1 = \frac{-8 + 5(1.84) + 3.3978}{5}$$
$$= 0.9196$$

$$x_2 = \frac{-4 - 0.9196 - 3.3978}{-4}$$
$$= 2.0794$$

$$x_3 = \frac{-18 + 2(0.9196) - 2.0794}{-6}$$
$$= 3.0400$$

4th Iteration

$$x_1 = \frac{-8 + 5(2.0794) + 3.0400}{5}$$
$$= 1.0874$$

$$x_2 = \frac{-4 - 1.0874 - 3.0400}{-4}$$
$$= 2.0319$$

$$x_3 = \frac{-18 + 2(1.0874) - 2.0319}{-6} = 2.9762$$

Gauss Seidal Method gives better approximation to the exact solution.

3 (a)

(1) :- $f(x) = x^2 \cos(x) + \sin(x)$

By using Regula falsi method :-

$f(x) = x^2 \cos(x) + \sin(x)$

let,

$x_0 = 1 \qquad x_1 = 2$

$f(x_0) = 1.38 \qquad f(x_1) = -0.755$

So that,

Root lies between x_0 and x_1

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \times f(x_0)$$

$$= 1 - \frac{2-1}{-0.755 - 1.38} \times (1.38)$$

$= 1.65$

$f(x_2) = 1.65^2 \cos(1.65) + \sin(1.65)$

$= 0.781$

The root lies between x_2 and x_1

taking,

$x_0 = 1.65 \qquad x_1 = 2$

$f(x_0) = 0.781 \qquad f(x_1) = -0.755$

we get,

$$x_3 = 1.65 - \frac{2 - 1.65}{-0.755 - 0.781} \times 0.781$$

$= 1.83$

$f(x_3) = 1.83^2 \cos(1.83) + \sin(1.83)$

$= 0.108$

The root lies between x_3 and x_1

taking

$x_0 = 1.83$ $\quad\quad$ $x_1 = 2$

$f(x_0) = 0.108$ $\quad\quad$ $f(x_1) = -0.755$

we get,

$$x_4 = 1.83 - \frac{2 - 1.83}{-0.755 - 0.108} \times 0.108$$

$$= 1.85$$

$$f(x_4) = 1.85^2 \cos(1.85) + \sin(1.85)$$

$$= 0.018$$

The root lies between x_4 and x_1

taking,

$x_0 = 1.85$ $\quad\quad$ $x_1 = 2$

$f(x_0) = 0.018$ $\quad\quad$ $f(x_1) = -0.755$

$$x_5 = 1.85 - \frac{2 - 1.85}{-0.755 - 0.018} \times 0.018$$

$$= 1.85$$

Hence the root is 1.85

Q3 (a) Determine the Smallest roots of the following equation:

(iii) $f(x) = x^2 \cos(x) + \sin(x) = 0$

Since $f(1)$ is +ve and $f(2)$ is −ve, a root lies between ~~1~~ and 2.

∴ First approximation to the root is

Thus, $x_1 = \frac{1}{2}(1+2) = 1.5$

$f(x_1) = 1.5^2 \cos(1.5) + \sin(1.5)$
$= 1.16$ i.e. +ve

∴ The root lies between x_1 and 2.
Thus the second approximation to the root is

$x_2 = \frac{1}{2}(2+1.5) = 1.75$

Thus,
$f(x_2) = 1.75^2 \cos(1.75) + \sin(1.75)$
$= 0.438$ +ve

∴ Thus root lies between x_2 and a (2)
the third approximation to the root is

$x_3 = \frac{1.75+2}{2} = 1.88$

Thus,
$f(x_3) = 1.88^2 \cos(1.88) + \sin(1.88)$
$= -0.123$ −ve

Thus, root lies between x_2 and x_3
the fourth approximation to the root is,

$$x_4 = \frac{1}{2}(1.88+1.75)$$
$$= 1.82$$

Thus

$$f(x_4) = 1.82^2 \cos(1.82) + \sin(1.82)$$
$$= 0.152 \text{ +ve}$$

The root lies between x_4 and x_3
thus fifth approximation to the root is

$$x_5 = \frac{1}{2}[1.82 + 1.88]$$
$$= 1.85$$

thus,

$$f(x_5) = 1.85^2 \cos(1.85) + \sin(1.85)$$
$$= 0.018 \text{ +ve}$$

The root lies between x_5 and x_3
thus 6th approximation to the root is

$$x_6 = \frac{1}{2}[1.85 + 1.88]$$
$$= 1.87$$

Thus,

$$f(x_6) = 1.87^2 \cos 1.87 + \sin 1.87$$
$$- 0.075 \text{ -ve}$$

The root lies between x_6 and x_5

$$x_7 = \frac{1}{2}[1.87 + 1.85]$$
$$= 1.86$$

Thus,

$f(x_7) = -0.028$ -ve

The root lies between x_7 and x_5

So that,

$$x_8 = \frac{1}{2}(1.86 + 1.85)$$

$$= 1.86$$

Hence the root is 1.86.

IV. Secant Method

$f(x) = x^2 \cos(x) + \sin(x)$

Taking the initial approximation

$x_0 = 1$ $\quad\quad x_1 = 2$

$f(x_0) = 1.38;$ $\quad\quad f(x_1) = -0.755$

Then, by secant method we have,

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \times f(x_1)$$

$$= 2 - \frac{2 - 1}{-0.755 - 1.38} \times -0.755$$

$$= 1.65$$

$$f(x_2) = 1.65^2 \cos(1.65) + \sin(1.65)$$

$$= 0.781$$

$\therefore$ Now

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_7)} \times f(x_2)$$

$$= 1.65 - \frac{1.65 - 2}{0.781 + 0.755} \times 0.781$$

$$= 1.83$$

Thus,

$$f(x_3) = 1.83^2 \cos(1.83) + \sin(1.83)$$
$$= 0.108$$

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} \times f(x_3)$$

$$= 1.83 - \frac{1.83 - 1.65}{0.108 - 0.781} \times 0.108$$

$$= 1.86$$

$$f(x_4) = 1.86^2 \cos(1.86) + \sin(1.86)$$
$$= -0.028$$

Now

$$x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} \times f(x_4)$$

$$= 1.86 - \frac{1.86 - 1.83}{-0.028 - 0.108} \times -0.028$$

$$= 1.85$$

$$f(x_5) = 1.85^2 \cos(1.85) + \sin(1.85)$$
$$= 0.018$$

$$X_6 = X_5 - \frac{X_5 - X_4}{f(X_5) - f(X_4)} \times f(X_5)$$

$$= 1.85 - \frac{1.85 - 1.86}{0.018 + 0.028} \times 0.018$$

$$X_6 = 1.85$$

Hence $X_6 = 1.85$ is correct root.

4(b):- Express $\nabla^3 f_1$ as a backward difference

$$y_p = y_0 + p\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_0$$

4(c):- Express $\Delta^3 f_1$ as a central difference.

$$y_p = y_0 + p\delta y_{1/2} + \frac{p(p-1)}{2!}\mu\delta^2 y_{1/2} + \frac{(p-\frac{1}{2})p(p-1)}{3!}\delta^3 y_{1/3}$$

4(d):-

I	x_i	y_i				
0	-1	16.8575				
			7.205			
1	0	24.0625		-14.703		
			-7.498		-8.302	
2	1	16.5650		-23.005		104.310
			-30.503		96.008	
3	2	-13.9375		73.003		-96.011
			42.500		-0.003	
4	3	28.5626		73.000		
			115.500			
5	4	144.0625				

5 (a) :-

(ii) Year (x) : 1971 1981 1991 2001 2011

Population(y): 112 132 158 189 226

By using Newton's forward formula

year (x)	Population (y)	Δ	Δ^2
1971	112		
		20	
1981	132		6
		26	
1991	158		5
		31	
2001	189		
		37	6
2011	226		

If we take $x_0 = 1991$, then $y_0 = 158$

$\Delta y_0 = 31$, $\Delta^2 y_0 = 6$.

Since $x = 1992$

$h = 10$

$$P = \frac{x - x_0}{h} = \frac{1992 - 1991}{10} = 0.1$$

Using Newton's forward formula,

$$y_{1992} = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0$$

$$= 158 + 0.1 \times 31 + \frac{0.1(0.1-1)}{\cancel{2}} \times \cancel{6}^{3}$$

$$= 160.83$$

Q:- 5(a)

x	y	∇	∇^2	∇^3	Δ^4
1971	112				
		20			
1981	132		6		
		26		-1	
1991	158		5		2
		31		1	
2001	189		6		
		37			
2011	226				

$$x = 1980 \qquad x_0 = 2011$$

$$p = \frac{x - x_0}{h} = \frac{1980 - 2011}{h} = -3.100$$

$$y_p = y_0 + p\, y \nabla + \frac{p(p+1)}{2!} y_0 \nabla^2 + \frac{p(p+1)(p+2)}{3!} y_0 \nabla^3$$

$$+ \frac{p(p+1)(p+2)(p+3)}{4!} y_0 \nabla^4$$

$$= 226 + -3.1 \times 37 + \frac{-3.1(-3.1+1)}{2} \times 6 +$$

$$- \frac{3.1(-3.1+1)(-3.1+2)}{3 \times 2} \times 1 +$$

$$\frac{-3.1(-3.1+1)(-3.1+2)(-3.1+3)}{4 \times 3 \times 2} \times 2$$

$$= \underline{129.6962}$$ Ans

Q:-5(b)

$f(1) = -32$, $f(4) = 08$, $f(5) = 52$, $f(7) = 167$

taking $x = 3$

By using Lagrange's interpolation formula,

$$\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times f(x_1)$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)(x_3-x_2)} \times f(x_3)$$

X	1	4	5	7
f(X)	-30	08	52	167

$$\frac{(3-4)(3-5)(3-7)}{(1-4)(1-5)(1-7)} \times -30 + \frac{(3-1)(3-5)(3-7)}{(4-1)(4-5)(4-7)} \times 08$$

$$\frac{(3-1)(3-4)(3-7)}{(5-1)(5-4)(5-7)} \times 52 + \frac{(3-1)(3-4)(3-5)}{(7-1)(7-4)(7-5)} \times 167$$

$= 44.333$

Similarly,

taking x

$$\frac{(x-4)(x-5)(x-7)}{-72} \times -30 + \frac{(x-1)(x-5)(x-7)}{9} \times 08$$

$$\frac{(x-1)(x-4)(x-7)}{8} \times 52 + \frac{(x-1)(x-4)(x-5)}{36} \times 167$$

Q:- 6

x	$f(x)$	Δ	Δ^2	Δ^3
76	5.3147			
		0.1199		
81	5.4346		0.0092	
		0.1291		-0.0391
86	5.5637		-0.0299	
		0.0992		
96	5.6629			

Choosing $x_0 = 76$

$x = 76$

$$P = \frac{x - x_0}{h} = \frac{76 - 76}{5} = 0$$

$$y'(x) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0\right]$$

$$= \frac{1}{5}\left[0.1199 - \frac{1}{2} \times 0.0092 + \frac{1}{3} \times -0.0391\right]$$

$$= \frac{1}{5}\left[0.1199 - 0.0046 - 0.0130\right]$$

$$0.0205$$

$$y''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0\right]$$

$$= \frac{1}{25}\left[0.0092 + 0.0391\right]$$

$$= 0.0019$$

Q:- 7(a):-

$$\int_{8.4}^{10.4} (5x + 4x^2 + 3)\, dx$$

(I) Rectangular Rule
taking $h = 1$
using Rectangular rule

$h\, [f(8.4) + f(10.4)]$

$h\, [327.240 + 487.640)$

$1\, [814.880]$

$= 814.880$

(II) By using Trapezodeal Rule
taking $h = 1$

$\frac{h}{2}\, [y_0 + y_n]$

$y_{8.4} = 327.240\ ,\ y_{10.4} = 487.640$

$\frac{h}{2}\, [\cancel{487}.\ 327.240 + 487.640]$

$\frac{1}{2}\, [814.880]$

$= 408440$

(III) By using Simpson's rule

$\frac{h}{3}(y_0 + y_n) = \frac{1}{3}(327.240 + 487.640)$

$= 971.627$

Q:- 8(a)

(1) $y' = (x-y)/2$, $y(0) = 1$,
find $y(0.8)$
taking $h = 0.2$

We have $y_{n+1} = y_n + hf_n$

$y_{0.2} \simeq y_1 = y_0 + (0.2) f_0$

$= 1 + 0.2\left[\frac{0-1}{2}\right]$

$= 1 + 0.2 \times -0.5$

$= 0.9$

$y_{0.4} \simeq y_2 = y_1 + (0.2) f_1$

$= 0.9 + 0.2\left[\frac{0.2-0.9}{2}\right]$

$= 0.83$

$y_{0.6} \simeq y_3 = y_2 + (0.2) f_2$

$= 0.83 + 0.2\left[\frac{0.4-0.83}{2}\right]$

$= 0.787$

$y_{0.8} \simeq y_4 = y_3 + 0.2\left[\frac{0.6-0.787}{2}\right]$

$= 0.768$

8 (a)(ii)

$y' = (x-y)/2 \quad y(0) = 1$

taking $h = 0.1$

We have $y_{n+1} = y_n + h f_n$

$y_{0.2} \simeq y_1 = y_0 + 0.1 \left[\frac{0-1}{2}\right]$

$= 1 + 0.1 \left[\frac{-1}{2}\right]$

$= 0.95$

$y_{0.4} \simeq y_2 = y_1 + 0.1 \left[\frac{0.2 - 0.95}{2}\right]$

$= 0.95 + 0.1 \left[\frac{0.2 - 0.95}{2}\right]$

$= 0.913$

$y_{0.6} \simeq y_3 = y_2 + 0.1 \left(\frac{0.4 - 0.913}{2}\right)$

$= 0.887$

$y_{0.8} \simeq y_4 = y_3 + 0.1 \left[\frac{0.6 - 0.887}{2}\right]$

$= 0.887 + 0.1 \left[\frac{0.6 - 0.887}{2}\right]$

$= 0.873$

8 (a) (ii)

taking $h = 0.1$

$y' = (x-y)/2 \quad y(0) = 1$

We have $y_{n+1} = y_n + hf_n$

$y_{0.2} \simeq y_1 = y_0 + 0.1 f_0$

$= 1 + 0.1 [0.2 - 0.$