BCS 012 Solved assignment july 2017 and January 2018 session

Q.1
A.1

$$0 = -4 + 4k + 6k - 4k^2 - 16k + 8 - 4k^2 + 2k$$

$$0 = -4k^2 - 4k^2 + 10k - 16k + 2k + 4$$

$$= -8k^2 + 12k - 16k + 4$$

$$= -8k^2 - 4k + 4$$

$$0 = 8k^2 + 4k - 4$$

$$0 = 2k^2 + k - 1$$

$$0 = 2k^2 + 2k - k - 1$$

$$0 = 2k(k+1) - 1(k+1)$$

$$0 = (k+1)(2k-1)$$

$$k + 1 = 0 \qquad\qquad 2k - 1 = 0$$

$$k = -1 \qquad\qquad k = \frac{1}{2}$$

Q.2
A.2

$$A = \begin{pmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ and } B = \begin{pmatrix} 14 \\ 4 \\ 0 \end{pmatrix}$$

The cofactors of $|A|$ are

$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} = -3$ $\quad A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} = 9$

and $A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 5.$

$\therefore\ |A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = (3)(-3) + 4(9) + 7(5) = 62.$

Since $|A| \neq 0$, A is non−singular (invertible). Its remaining cofactors are

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 4 & 7 \\ 2 & -3 \end{vmatrix} = 26, \quad A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 7 \\ 1 & -3 \end{vmatrix} = -16,$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = -2, \quad A_{31} = (-1)^{3+1}\begin{vmatrix} 4 & 7 \\ -1 & -3 \end{vmatrix} = 19,$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 7 \\ 2 & 3 \end{vmatrix} = 5, \qquad A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -11$$

The adjoint of matrix A is given by

$$\operatorname{adj} A = \begin{pmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{pmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\operatorname{adj}A = \frac{1}{62}\begin{pmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{pmatrix}$$

$$\text{Also, } X = A^{-1}B = \frac{1}{62}\begin{pmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{pmatrix}\begin{pmatrix} 14 \\ 4 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{62}\begin{pmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{pmatrix}\begin{pmatrix} 14 \\ 4 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{62}\begin{pmatrix} -42 + 104 \\ 126 - 64 \\ 70 - 8 \end{pmatrix} = \frac{1}{62}\begin{pmatrix} 62 \\ 62 \\ 62 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Hence $x = 1$, $y = 1$, $z = 1$ is the required solution.

Q.3

A.3 Assume n = k;
and solve it..

We shall now show that the true of P_k implies the truth of P_{k+1} where P_{k+1} is

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.+\frac{1}{k(k+1)}=\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$$

$$\text{LHS of (1)} = \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$$

$$=\frac{k}{(k+1)}+\frac{1}{(k+1)(k+2)} \qquad \text{[induction assumption]}$$

$$=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$$

= RHS of (1)

Q.4
A.4

Let $39 + 13\sqrt{3}$ be the sum to n terms of the given GP, that is,

$S_n = 39 + 13\sqrt{3}$

$$\Rightarrow \quad \frac{a(r^n-1)}{r-1} = 39 + 13\sqrt{3} \quad \Rightarrow \frac{(\sqrt{3})\left[(\sqrt{3})^n - 1\right]}{\sqrt{3}-1}$$

$$= 13\sqrt{3}\,(\sqrt{3}+1)$$

$$\Rightarrow \qquad 3^{n/2} = 1 + 13(3-1) = 1 + 26 = 27 = 3^3 \Rightarrow n/2 = 3 \text{ or } n = 6.$$

Thus, 6 terms of $\sqrt{3}, 3, 3\sqrt{3}$ ... ...
are required to obtain a sum of $39 + 13\sqrt{13}$.

Q.5
A.5

Differentiating both sides with respect to x, we get $\frac{dy}{dx} = \frac{d}{dx}\left(ae^{mx} + be^{-mx}\right)$
$= ame^{mx} - bme^{-mx}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(ame^{mx} - bme^{-mx}\right)$$

$$= am^2e^{mx} - bm(-m)e^{-mx}$$

$$= am^2e^{mx} + bm^2e^{-mx}$$

$$= m^2\left(ae^{mx} + be^{-mx}\right)$$

$$= m^2y$$

Q.6
A.6

$$\frac{x}{(x+1)(2x-1)} = \frac{A}{x+1} + \frac{B}{2x-1}$$

$\Rightarrow x = A(2x-1) + B(x+1)$

Put $x = ½$ and -1 to obtain

$$\frac{1}{2} = B\left(\frac{1}{2}+1\right) \Rightarrow B = \frac{1}{3}$$

$$-1 = A(-3) \Rightarrow A = \frac{1}{3}$$

Thus,

$$\int \frac{x}{(x+1)(2x-1)}\,dx = \frac{1}{3}\int \frac{dx}{(x+1)} + \frac{1}{3}\int \frac{dx}{(2x-1)}$$

$$= \frac{1}{3}\log|x+1| + \frac{1}{3}\cdot\frac{1}{2}\log|2x-1| + c$$

$$= \frac{1}{3}\log|x+1| + \frac{1}{6}\log|2x-1| + c$$

Q.7
A.7

(i) As $1+\omega+\omega^2=0$, we get

$1+\omega=-\omega^2$ and $1+\omega^2=-\omega$

Thus,

$$(1+\omega)^2-(1+\omega^2)^3+\omega^2$$

$$=(-\omega^2)^2-(-\omega)^3+\omega^2$$

$$=\omega^4+\omega^3+\omega^2=\omega^3\omega+1+\omega^2$$

$$=\omega+1+\omega^2=0$$

Q.8
A.8

$\alpha+\beta=3/2$ and $\alpha\beta=-5/2$

We are to form a quadratic equation whose roots are α^2, β^2.

Let S = Sum of roots = $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$.

$$=\left(\frac{3}{2}\right)^2-2\left(-\frac{5}{2}\right)=\frac{9}{4}+5=\frac{29}{4}$$

P = Product of roots = $\alpha^2\beta^2=(\alpha\beta)^2=\left(-\frac{5}{2}\right)^2=\frac{25}{4}$.

Putting values of S and P in $x^2-Sx+P=0$, the required equation is

$$x^2-(29/4)x+25/4=0 \text{ or } 4x^2-29x+25=0.$$

Q.9
A.9

$$\frac{3(x-2)}{5} \le \frac{5(2-x)}{3} \Leftrightarrow 3[3(x-2] \le 5[5(2-x)]$$

$$\Leftrightarrow \quad 9x - 18 \le 50 - 25x$$

$$\Leftrightarrow \quad 9x + 25x \le 50 + 18$$

$$\Leftrightarrow \quad 34x \le 68 \text{ or } x \le 2$$

$$\therefore \text{ Solution set is } \{x \mid x \le 2\} = (-\infty, 2]$$

The graph of the inequality is

Q.10
A.10

$$\text{Thus, } V = \frac{4}{3}\pi r^3$$

Differentiating both the sides of this equation with respect to t, we get

$$\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}.$$

Note that have used the chain rule on the right hand side

It is given that $\frac{dV}{dt} = 900$ *and* $r = 15$. Thus,

$$900 = 4\pi\,(15)^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{900}{4\pi\,(225)} = \frac{1}{\pi}\ cm/\,sec.$$

Thus, the radius is increasing at the rate of $= \frac{1}{\pi}\ cm/\,sec.$

Q.11
A.11

$x^2 = x \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1.$

Thus, the two curves intersect in (0,0) and (1,1). See Fig 4.8

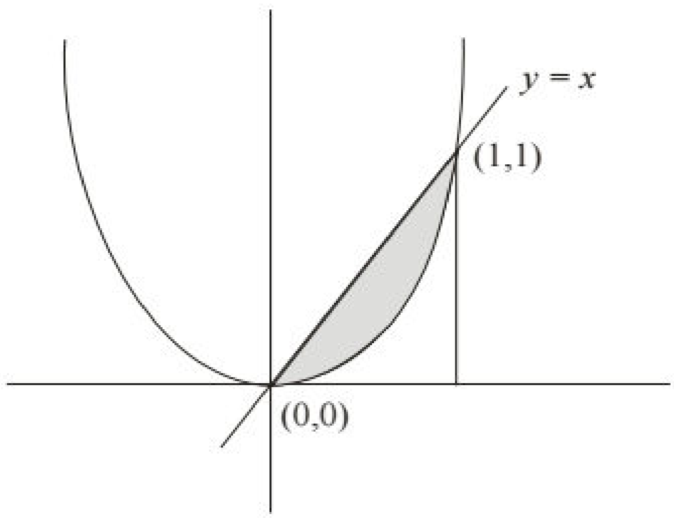
Required area

$= \int_0^1 (x - x^2)dx$

$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$

$= \left(\frac{1}{2} - \frac{1}{3}\right)$ sq. units

$= \frac{1}{6}$ sq. units.



Q.12
A.12
Suppose Investors invests ` x in saving certificate and ` y in National Savings Bonds.
As he has just ` 12000 to invest, we must have x + y ≤ 12000.
Also, as he has to invest at least ` 2000 in savings certificate x ≥ 2000.
Next, as he must invest at least Rs. 4000 in National Savings Certificate y ≥ 4000. Yearly income from saving certificate = ` = 0.08x and from National Savings Bonds = ` = Rs. 0.1y
His total income is ` P where

P = 0.08x + 0.1y
Thus, the linear programming problem is
Maximise

subject to
x + y ≤ 12000 [Total Money Constraint]
x ≥ 2000 [Savings Certificate Constraint]
y ≥ 4000 [National Savings Bonds Constraint]
x ≥ 0, y ≥ 0 [Non-negativity Constraint]
However, note that the constraints x ≥ 0, y ≥0, are redundant in view of

x ≥ 2000 and y ≥ 4000.

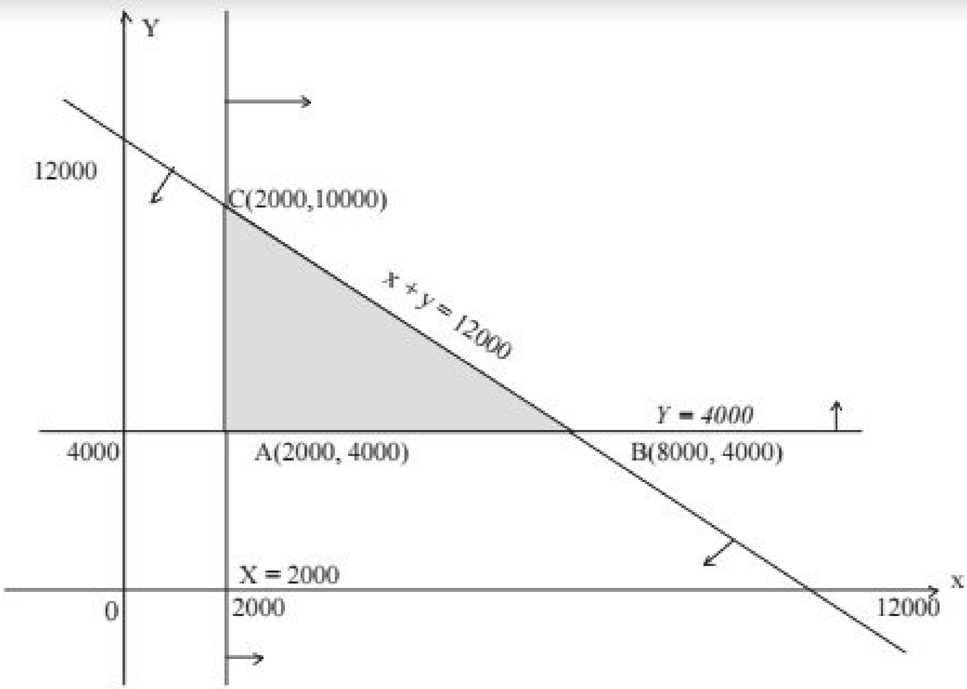


Figure 8

We now calculate the profit at the corner points of the feasible region.

We have

$$P(A) = P(2000,4000) = (0.08)(2000) + (0.1)(4000)$$
$$= 160 + 400 = 560$$

$$P(B) = P(8000,4000) = (0.08)(2000) + (0.1)(4000)$$
$$= 640 + 400 = 1040$$

$$P(C) = P(2000,10000) = (0.08)(2000) + (0.1)(10000)$$
$$= 160 + 1000 = 1160.$$

$P(C) = P(2000,10000) = (0.08)(2000) + (0.1)(10000)$
$= 160 + 1000 = 1160.$

Thus, she must invest ₹ 2000 in Savings certificate and ₹ 10000 in National Savings Bonds in order to earn maximum income.

Q.13
A.13

$$A = \begin{bmatrix} 2 & 5 & -3 & -4 \\ 4 & 7 & -4 & -3 \\ 6 & 9 & -5 & 2 \\ 0 & -9 & 6 & 15 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 5 & -3 & -4 \\ 0 & -3 & 2 & 5 \\ 0 & -6 & 4 & 14 \\ 0 & -9 & 6 & 15 \end{bmatrix} \quad (\text{by } R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1)$$

$$\sim \begin{bmatrix} 2 & 5 & -3 & -4 \\ 0 & -3 & 2 & 5 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{by } R_3 \to R_3 - 2R_2,\ R_4 \to R_4 - 3R_2)$$

$= B$

Clearly, rank of B cannot B 4; as $|B| = 0$.

Also, $\begin{bmatrix} 2 & 5 & -4 \\ 0 & -3 & 5 \\ 0 & 0 & 4 \end{bmatrix}$ is a square submatrix

of order 3 of B and $\begin{vmatrix} 2 & 5 & -4 \\ 0 & -3 & 5 \\ 0 & 0 & 4 \end{vmatrix} = 2 \times (-3) \times 4 = -24 \neq 0$

So, rank of matrix B is 3.

Hence rank of matrix A = 3.

Q.14
A.14

$$A = I_3 A$$

$$\begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Again, applying $R_2 \rightarrow R_2 - 2R_1$ and , $R_3 \rightarrow R_3 + R_1$, we get

$$\begin{bmatrix} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A$$

Again, applying $R_3 \rightarrow R_3 + R_2$, we have

$$\begin{bmatrix} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A$$

Since, we have obtained a row of zeros on the L.H.S., we see that A cannot be reduced to an identity matrix. Thus, A is not invertible, Infact, note that A is a singular matrix as $|A| = 0$.

Q.15

A.15

We are given that $m\, a_m = n\, a_n$

$\Rightarrow \quad m[a + (m-1)d] = n[a + (n-1)d]$

$\Rightarrow \quad m[a + md - d] = n[a + nd - d]$

$\Rightarrow \quad m[\alpha + md] = n[\alpha + nd]$ *where* $\alpha = a - d$

$\Rightarrow \quad m\alpha + m^2 d = n\alpha + n^2 d \Rightarrow (m-n)\alpha + (m^2 - n^2)d = 0$

$\Rightarrow \quad (m-n)[\alpha + (m+n)d] = 0 \quad \Rightarrow \alpha + (m+n)d = 0$

$\Rightarrow \quad a + (m+n-1)d = 0 \qquad [\because \alpha = a - d]$

Left hand side is nothing but the $(m+n)th$ term of the A.P.

Q.16
A.16

: Let $f(x) = \frac{|x|}{x}$, $x \neq 0$.

Since $|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$

$\therefore f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$

So, $\lim_{x \to 0+} f(x) = \lim_{x \to 0}(1) = 1$ and

$\lim_{x \to 0-} f(x) = \lim_{x \to 0}(-1) = -1$

Thus $\lim_{x \to 0} f(x)$ does not exist.

$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

To show that f is continuous at $x = 0$, it is sufficient to show that

$\lim_{x \to 0-} f(x) = \lim_{x \to 0+} (x) = f(0)$ and

We have

$\lim_{x \to 0-} f(x) = \lim_{h \to 0+} f(0 - h) = \lim_{h \to 0+} f(-h)$

$= \lim_{h \to 0+} -(-h)$

$= \lim_{h \to 0+} h = 0$

and $\lim_{x \to 0+} f(x) = \lim_{h \to 0+} f(0 + h) = \lim_{h \to 0+} f(h)$

$= \lim_{h \to 0+} (h) = 0.$

Thus, $\lim_{h \to 0-} f(x) = \lim_{h \to 0+} f(x) = 0$

Also, $f(0) = 0$

Therefore, $\lim_{x \to 0+} f(x) = 0 = f(0)$

Hence, f is continuous at $x = 0$.